CALCULATION OF PRINCIPAL CHARACTERISTICS OF TURBULENT STREAMS IN A STATE OF STRUCTURAL EQUILIBRIUM

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A review of articles on the study of turbulent streams having transverse displacement, in which a turbulent energy balance equation is used, is contained in [1]. Levin [2] proposed a certain development of Rotta's method [3] making it possible to determine the characteristics of the average flow and the radial distribution of pulsation magnitudes. However, in this article the scale of the turbulence (the quantity l) was given as an empirical function of the coordinates. At the same time it is clear that the distribution of the turbulence scale depends on the conditions of the problem. A special differential equation proposed in [4,5] describing the variation in time and space of the quantity l has the drawback that in deriving this equation it is necessary to invoke additional hypotheses which are difficult to test experimentally. In the present article, along with the velocity of the average flow, the pressure, and the pulsation magnitudes, the scale of the turbulence is considered as an important characteristic of the stream, determined by the reference system which consists of the Reynolds equations, continuity equations, and equations for the component of the Reynolds stress tensor. Rotta's approximate semiempirical relations and an experimental relation for the single-point correlation coefficient between the turbulent pulsations in velocity are used for closure of the system obtained. An approximate calculation is given for the principal average and pulsation characteristics of the flow for the region of the stream where the turbulence is in a state of structural equilibrium [6]. A comparison of the calculated and experimental data is presented.

For a broad range of flows having transverse displacement, including flow in a boundary layer at rough and smooth walls [7], streams in pipes and channels [8, 9], flow in a short-range wake, and jets [10, 11], a similarity can be observed between the distribution of turbulent stress and the distribution of the corresponding turbulence intensities. The linear relation between the turbulent stress of friction and the turbulent kinetic energy was used by Bradshaw [12] in calculating the friction in a boundary layer and by Lee and Harsha [13] in analyzing jet streams. An experimental confirmation of this hypothesis in a broad range of variation of the flow conditions is presented in [14]. It is indicated in [15] that in rotating streams the ratio of the turbulent stress to the corresponding components of the turbulence intensity is also constant in cross section and approximately equal to 0.4.

If we introduce the correlation coefficient

$$k_{uv} = \langle uv\rangle / \sqrt{\langle u^2\rangle \langle v^2\rangle} \tag{1}$$

then outside the viscous sublayer and the axial zone this coefficient is constant and equal to ~0.42 (the points 1 in Fig. 1 are plotted from the data of [7], points 2 from [8], 3 from [9], 4 from [10], and 5 from [11]). It must be noted that the experiments conducted in the case of uniform distortion of homogeneous turbulence [6] showed that the maximum possible value of this coefficient is also equal to 0.42 ([6], p. 377). The constancy of the correlation coefficient can serve as an indication that in this region the turbulence is in a state of structural equilibrium [6]. Townsend indicates that the establishment of an equilibrium struc-

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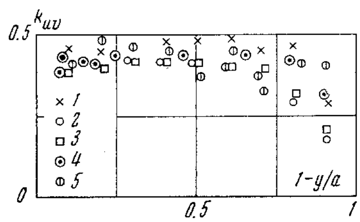

Fig. 1

ture can formally be represented as the establishment of some equilibrium between the orienting effect of the distortion on vortices and the general tendency of turbulent flow to become isotropic.

Taking advantage of this fact, let us examine as an example the calculation of the principal characteristics of turbulent streams in a cylindrical pipe and a flat channel. For the components of the Reynolds stress tensor we have the following system of equations in Cartesian coordinates [1] (in tensor notation):

$$\frac{\partial\langle u_iu_j\rangle}{\partial t}+U_k\frac{\partial\langle u_iu_j\rangle}{\partial x_k}+\langle u_ku_j\rangle\frac{\partial U_i}{\partial x_k}+\langle u_ku_i\rangle\frac{\partial U_j}{\partial x_k}-$$

$$-\frac{1}{\rho}\left\langle p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right\rangle+\frac{\partial}{\partial x_k}\left[-\nu\frac{\partial\langle u_iu_j\rangle}{\partial x_k}+\langle u_iu_ju_k\rangle+\left\langle\frac{p}{\rho}\left(\delta_{jk}u_i+\delta_{ik}u_j\right)\right\rangle\right]+2\nu\left\langle\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle=0 \qquad (2)$$

where p is the pulsation pressure and δ_{ij} is the Kronecker symbol (i, j = 1, 2, 3).

An analysis of experimental data [8] shows that in the region of flow $0.3 < y/a < 0.9$, where the correlation coefficient k_{uv} is constant, convective diffusion (due to turbulence) and viscous diffusion can be neglected.

On the example of [2] we take the following expression for the term expressing the exchange of energy between the three components of the pulsations:

$$\frac{1}{\rho}\left\langle p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right\rangle=-k\frac{E^{1/2}}{l}\left(\langle u_iu_j\rangle-\frac{2}{3}\delta_{ij}E\right) \qquad (3)$$

while for the dissipative term we take the interpolation equation

$$2\nu\left\langle\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_u}\right\rangle=\nu c_1\frac{\langle u_iu_j\rangle}{l^2}+\delta_{ij}\frac{2}{3}\frac{cE^{3/2}}{l} \qquad (4)$$

$$2E=\sum_i\langle u_i^2\rangle$$

Here k, c, and c_1 are constant coefficients.

The use of the dependence (4) in a particular case of isotropic turbulence gives certain laws of degeneration (in the initial and final stages) of the energy of the turbulence. Actually, the sum of the first three equations of system (2) gives a balance equation for the total pulsation energy, which in the case of isotropic turbulence and the application of Eq. (4) has the form

$$\frac{\partial E}{\partial t}+c\frac{E^{3/2}}{l}+c_1\nu\frac{E}{l^2}=0 \qquad (5)$$

The process of degeneration of isotropic turbulence in the initial stage is determined for the most part by the decay of energy-containing vortices; in the final stage the effect of viscosity predominated over inertial effects. It is shown in [16] that if the value $(\nu t)^{1/2}$ is chosen as the characteristic length, then in the case of very small Reynolds numbers (final stage of degeneration) the energetic spectrum conserves similarity. This same characteristic length can also be chosen for the region of energy-containing vortices since in this case the relation $\varepsilon t^2/\nu$ = const is satisfied (ε is the energy dissipation per unit mass) ([16], p. 245). Substituting the value $\sqrt{\nu t}$ in place of l in Eq. (5) we obtain $E = (\nu/c^2)t^{-1}$ for the initial stage of the process of degeneration of isotropic turbulence, while for the final stage we obtain $E \sim t^{-c_1}$. These simple laws of the degeneration of isotropic turbulence have repeatedly been noted by different investigators (see, for example [16]). According to the experiments of Batchelor and Townsend [17] the constants c and c_1 have the values 0.155 and 2.5, respectively.

For closure of system (2) we draw on the equation of motion for the average flow which is uniform longitudinally:

$$\langle uv\rangle-\nu\frac{dU_1}{dy}=-\nu V_*^2\frac{y}{a}\quad\left(V_*^2=-\nu\left(\frac{dU_1}{dy}\right)_{y=a}\right) \qquad (6)$$

and substitute the turbulent friction according to Eq. (1).

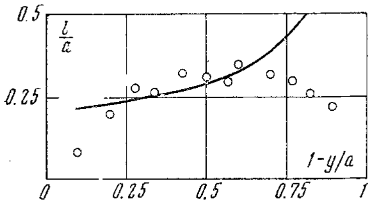

Fig. 2

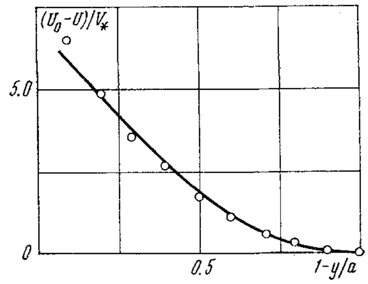

Fig. 3

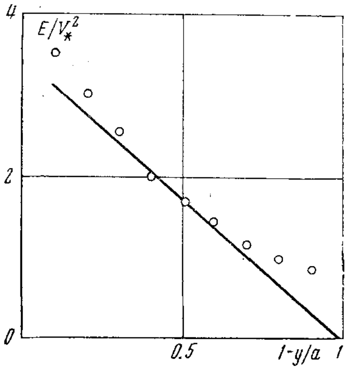

Fig. 4

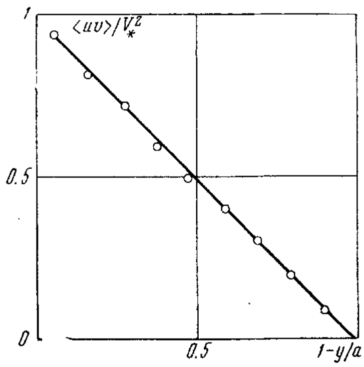

Fig. 5

In dimensionless form Eqs. (6) and (2), taking (3) and (4) into account, have the form [2]

$$\frac{k_{uv}\sqrt{\langle u^2\rangle}\sqrt{\langle v^2\rangle}}{E}=\frac{\mathrm{Re}_l}{\mathrm{Re}_E^2}+\frac{V_*^2 y}{Ea}$$

$$\frac{k_{uv}\sqrt{\langle u^2\rangle}\sqrt{\langle v^2\rangle}}{E}\mathrm{Re}_l+c\,\mathrm{Re}_E+c_1=0,\quad \frac{\langle v^2\rangle}{E}(k\,\mathrm{Re}_l+c_1)-\frac{2}{3}(k-c)\,\mathrm{Re}_E=0$$

$$\frac{\langle w^2\rangle}{E}(k\,\mathrm{Re}_E+c_1)-\frac{2}{3}(k-c)\,\mathrm{Re}_E=0$$

$$\frac{\langle v^2\rangle}{E}\mathrm{Re}_l+\frac{k_{uv}\sqrt{\langle u^2\rangle}\sqrt{\langle v^2\rangle}}{E}(k\,\mathrm{Re}_E+c_1)=0$$

$$\frac{\langle vw\rangle}{E}\mathrm{Re}_l+\frac{\langle uw\rangle}{E}(k\,\mathrm{Re}_E+c_1)=0$$

$$\frac{\langle vw\rangle}{E}(k\,\mathrm{Re}_E+c_1)=0$$

$$\left(\mathrm{Re}_E=l\sqrt{E}/\nu,\quad \mathrm{Re}_l=\frac{l^2}{\nu}\frac{dU_1}{dy}\right) \tag{7}$$

Solving system (7) we find

$$\langle uw\rangle=\langle vw\rangle=0$$

$$\frac{\langle uv\rangle}{V_*^2}=\left(1-\frac{\mathrm{Re}_l^2}{c\,\mathrm{Re}_E^3+c_1\mathrm{Re}_E^2+\mathrm{Re}_l^2}\right)\frac{y}{a}$$

$$\frac{\sqrt{\langle v^2\rangle}}{V_*}=\frac{\sqrt{\langle w^2\rangle}}{V_*}=\left[\frac{2(c-k)\,\mathrm{Re}_E^3\,\mathrm{Re}_l^2}{3(k\,\mathrm{Re}_E+c_1)(c\,\mathrm{Re}_E^3+c_1\mathrm{Re}_E^2\,\mathrm{Re}_l^2)}\right]^{1/2}\left(\frac{y}{a}\right)^{1/2}$$

$$\frac{\sqrt{\langle u^2\rangle}}{V_*}=-\frac{\mathrm{Re}_l}{k_{uv}}\left[\frac{2(c-k)\,\mathrm{Re}_E^3\,\mathrm{Re}_l}{3(k\,\mathrm{Re}_E+c_1)^3(c\,\mathrm{Re}_E^3+c_1\mathrm{Re}_E^2+\mathrm{Re}_l^2)}\right]^{1/2}\left(\frac{y}{a}\right)^{1/2}$$

$$\frac{U_0-U_1}{V_*}=\frac{\mathrm{Re}_l^2\,\mathrm{Re}_*}{2(c\,\mathrm{Re}_E^3+c_1\mathrm{Re}_E^2+\mathrm{Re}_l^2)}\left(\frac{y}{a}\right)^2$$

$$\frac{l}{a}=\left[-\frac{c\,\mathrm{Re}_E^3+c_1\mathrm{Re}_E^2+\mathrm{Re}_l^2}{\mathrm{Re}_*^2\,\mathrm{Re}_l}\right]^{1/2}\left(\frac{a}{y}\right)^{1/2}$$

$$\mathrm{Re}_*=\frac{V_*a}{\nu},\quad \mathrm{Re}_E=\frac{c_1(1-2k_{uv}^2)}{2kk_{uv}^2-c-4(k-c)k_{uv}^2/3}$$

$$\mathrm{Re}_l=-(k\,\mathrm{Re}_E+c_1)\left[\frac{3(c\,\mathrm{Re}_E+c_1)}{2(k-c)\,\mathrm{Re}_E}\right]^{1/2} \tag{8}$$

A comparison of the results of a calculation by Eq. (8) with experimental data is presented in Figs. 2-5 ($c = 0.155$, $c_1 = 2.5$, $k = 1.02$, $k_{uv} = 0.42$) (the solid curve is calculated). The value of the coefficient k was determined from a comparison of the experimental [8] and calculated profiles of the average velocity. The scale distribution taken from [5] ($V_*/U_0 = 0.037$) is shown by circles in Fig. 2. In Figs. 3-5 the circles represent Laufer's data [8], $V_*/U_0 = 0.035$.

It is seen from the graphs that such an approach can be used to determine the principal and pulsation characteristics of streams in those regions where the turbulence is in a state of structural equilibrium.

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